Supplementary Material for Correspondences of Persistent Feature Points on Near-Isometric Surfaces

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A Threshold Relaxation

This appendix discusses the issue of threshold relaxation of τ for noisy data. We use the following heuristic method to relax τ. Let S and $\widetilde{S}$ denote two near-isometric models, such that for any two pairs of corresponding points $\mathbf{c}_k, \widetilde{\mathbf{c}}_i$ and $\mathbf{c}_l, \widetilde{\mathbf{c}}_j$, the geodesic distortion is bounded by a threshold as follows $d(i,j,k,l) := \min\left(g\left(\mathbf{c}_k, \mathbf{c}_l\right)/g\left(\widetilde{\mathbf{c}}_i, \widetilde{\mathbf{c}}_j\right), g\left(\widetilde{\mathbf{c}}_i, \widetilde{\mathbf{c}}_j\right)/g\left(\mathbf{c}_k, \mathbf{c}_l\right)\right) \geq \tau$.

Assume that only shape $\widetilde{S}$ is corrupted by noise. Consider two points $\widetilde{\mathbf{c}}_i$ and $\widetilde{\mathbf{c}}_j$ on $\widetilde{S}$ and their corresponding noisy copies $\widetilde{\mathbf{c}}_i^*$ and $\widetilde{\mathbf{c}}_j^*$. In the following, we assume that for every point the length of $\widetilde{\mathbf{c}}_i - \widetilde{\mathbf{c}}_i^*$ is less than a constant δ. With this assumption, the length of any edge on the geodesic path between $\widetilde{\mathbf{c}}_i$ and $\widetilde{\mathbf{c}}_j$ changes (either increases or decreases) by at most 2δ and hence, the total change in the geodesic $g(\widetilde{\mathbf{c}}_i, \widetilde{\mathbf{c}}_j)$ is at most $2m\delta$, where m is the number of edges on the geodesic path from $\widetilde{\mathbf{c}}_i$ to $\widetilde{\mathbf{c}}_j$.

We will use this bound to derive a relaxation of the threshold τ. We are only interested in changes that decrease the value of $d(i,j,k,l)$, since changes that increase $d(i,j,k,l)$ make the two shapes more isometric and do not cause problems. These changes occur when $d(i,j,k,l) = g\left(\widetilde{\mathbf{c}}_i, \widetilde{\mathbf{c}}_j\right)/g\left(\mathbf{c}_k, \mathbf{c}_l\right)$ and $g\left(\widetilde{\mathbf{c}}_i, \widetilde{\mathbf{c}}_j\right)$ decreases and when $d(i,j,k,l) = g\left(\mathbf{c}_k, \mathbf{c}_l\right)/g\left(\widetilde{\mathbf{c}}_i, \widetilde{\mathbf{c}}_j\right)$ and $g\left(\widetilde{\mathbf{c}}_i, \widetilde{\mathbf{c}}_j\right)$ increases.

For the first case, we use that $g\left(\widetilde{\mathbf{c}}_i^*, \widetilde{\mathbf{c}}_j^*\right) \geq g\left(\widetilde{\mathbf{c}}_i, \widetilde{\mathbf{c}}_j\right) - 2m\delta$. Arithmetic manipulations yield $d(i,j,k^*,l^*) \geq d(i,j,k,l) - 2m\delta/g\left(\mathbf{c}_k, \mathbf{c}_l\right)$. For the second case, we use that $g\left(\widetilde{\mathbf{c}}_i^*, \widetilde{\mathbf{c}}_j^*\right) \leq g\left(\widetilde{\mathbf{c}}_i, \widetilde{\mathbf{c}}_j\right) + 2m\delta$. Hence, $d(i,j,k^*,l^*) \geq d(i,j,k,l) - 2m\delta g\left(\mathbf{c}_k, \mathbf{c}_l\right) / \left(\left(g\left(\widetilde{\mathbf{c}}_i, \widetilde{\mathbf{c}}_j\right)\right)^2 - 4m^2\delta^2\right)$.

If we assume that S and $\widetilde{S}$ have the same uniform resolution, we know that the number m of edges on the geodesic path from $\widetilde{\mathbf{c}}_i$ to $\widetilde{\mathbf{c}}_j$ is similar to the number of edges on the geodesic path from $\mathbf{c}_k$ to $\mathbf{c}_l$ and that all edge lengths are similar. Let $|e|$ denote the mesh resolution in both meshes. For the analysis, we assume that on both shapes, a path with m edges has length $m|e|$. This allows the following simplifications: $d(i,j,k^*,l^*) \geq d(i,j,k,l) - 2\delta/|e|$ and $d(i,j,k^*,l^*) \geq d(i,j,k,l) - 2\delta|e|/\left(|e|^2 - 4\delta^2\right)$. Hence, for two meshes that have the same uniform resolution, by adjusting τ to

$$\tau^* = \tau - \max\left(\frac{2\delta}{|e|}, \frac{2\delta|e|}{|e|^2 - 4\delta^2}\right) = \tau - \frac{2\delta|e|}{|e|^2 - 4\delta^2}, \tag{1}$$

the inequality $d(i, j, k^*, l^*) \geq \tau^*$ holds for all pairs for which $d(i, j, k, l) \geq \tau$ holds for the surfaces without noise. However, this is a loose upper bound and it is possible that a pair of points satisfies τ^* even though the corresponding pair of non-noisy points does not satisfy τ.

We consider Gaussian noise with zero mean and standard deviation σ. With this model, we expect the length of the offset vector applied to $\widetilde{\mathbf{c}}_i$ to be less than or equal to σ with high probability (about 68%). We relax the thresold by heuristically setting $\delta = \sigma$ in Equation 1.

B Correspondence Errors

This appendix shows the correspondence errors $\mathcal{C}$ for different model classes degraded with different types of deformations and noise. The left sides of Figures 1–5 show $\mathcal{C}$ averaged over correspondences between all models of one object class (i.e. all deformed clean models of one class registered to the clean neutral model of the same object class). Note that overall, the quality of the correspondence does not degrade significantly as a function of the levels of different types of noise. The right sides of Figures 1–5 show $\mathcal{C}$ for the correspondences computed between pairs of models of different object classes. Note that as long as consistent results are obtained, $\mathcal{C}$ does not increase significantly as the level of non-isometry increases.

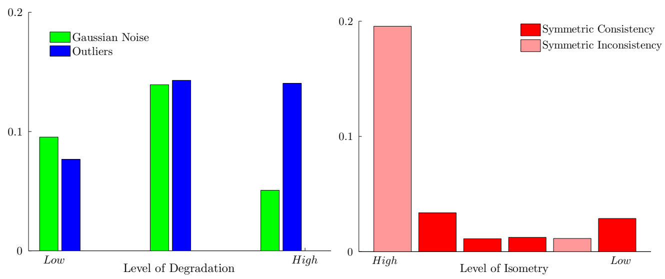

Figure 1. Correspondence errors $\mathcal{C}$ for the *David* model.

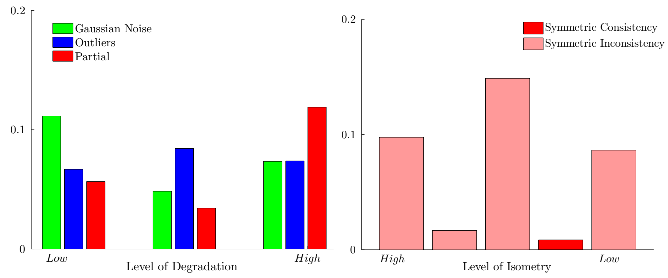

Figure 2. Correspondence errors $\mathcal{C}$ for the *Centaur* model.

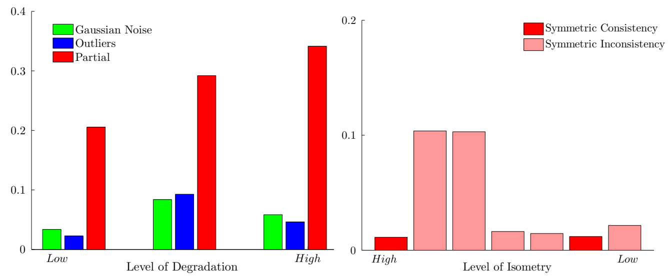

Figure 3. Correspondence errors $\mathcal{C}$ for the *Horse* model.

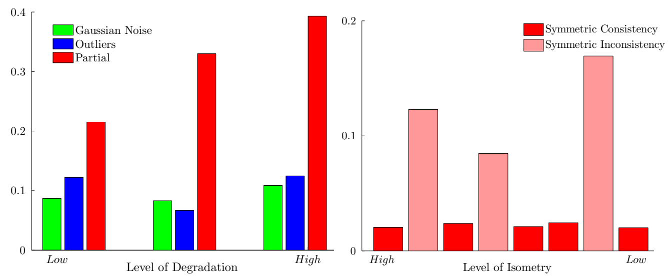

Figure 4. Correspondence errors $\mathcal{C}$ for the *Dog* model.

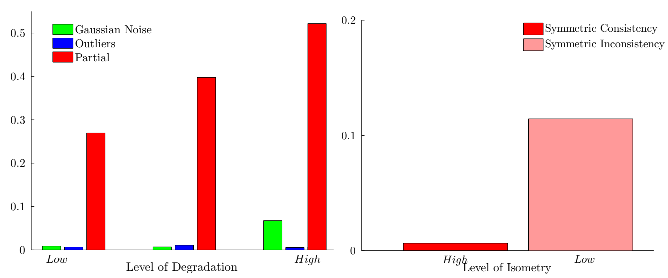

Figure 5. Correspondence errors $\mathcal{C}$ for the *Wolf* model.