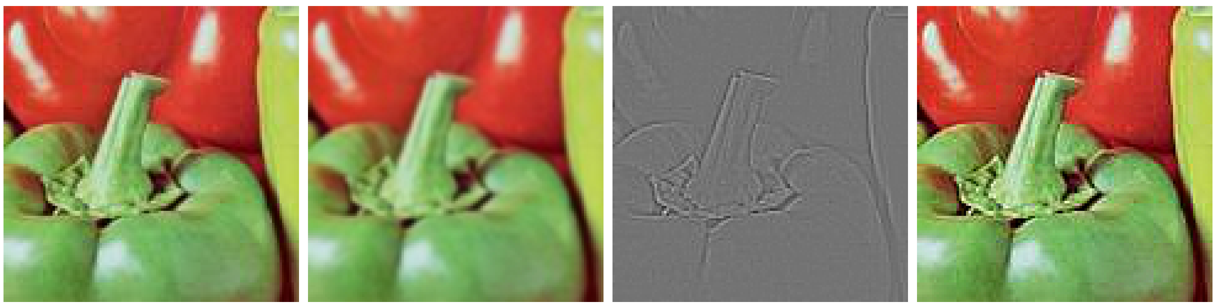
Exam December 3th, 2018

Duration: 3h. Lecture notes are authorized. Answers should be ***justified*** *briefly.*

1 Sharpening with the unsharp mask (4 points)



(a)	(b)	(c)	(d)
Input image	$\frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$	$\frac{1}{16}\begin{bmatrix} -1 & -2 & -1 \\ -2 & 12 & -2 \\ -1 & -2 & -1 \end{bmatrix}$	$\frac{1}{16}\begin{bmatrix} -1 & -2 & -1 \\ -2 & 28 & -2 \\ -1 & -2 & -1 \end{bmatrix}$

The above figure (1) shows results of convolutions of the input image with 3 filters whose masks are given below the images (note that an offset of 128 has been added to pixels in image (c) to ease visualisation). Filters were applied on the 3 color channels. The final image (d) is the sharpened image and the image (b) is the *unsharped* image.

1. What is the filter used in image (b) and what is it supposed to do ?
2. What is the relationship between (a), (b) and (c) ?
3. From the mask used in (d) deduce the principle of this sharpen filter.

2 Image Formation (6 points)

Assume we observe a square of edge e with a perspective projection of focal length f which center is at distance d from the square center.

1. Assume a finite focal length:
   (a) Does the projected square appear bigger or smaller when we increase f ? d ?
   (b) Is the projected square still a square when the observed square is in a plane parallel to the image plane ?
   (c) Can it be a square when the observed square is in a plane not parallel to the image plane ?

2. Same questions as above but with an infinite focal length (we increase only d in this case) ?
3. Assume again a finite focal length. The square edges are along two directions:
   (a) Assume further that one of the directions is in the image plane and the other not, do we observe a rectangle ?
   (b) These directions define vanishing points in the image plane. Could we see these points in the image plane ?

3 Histogram Transformation (5 points)

We want to transform the intensities of an image so that its cumulative distribution fits a given distribution $F^*(I^*)$ as shown in Figure (1). We consider the cumulative distributions of intensities, $F(I)$ and $F^*(I^*)$, and we are therefore looking for the transformation $I^* = T(I)$ (illustrated by the arrow in figure 1) such that the cumulative distribution $F^*(I^*)$ is the red curve in Figure (1).

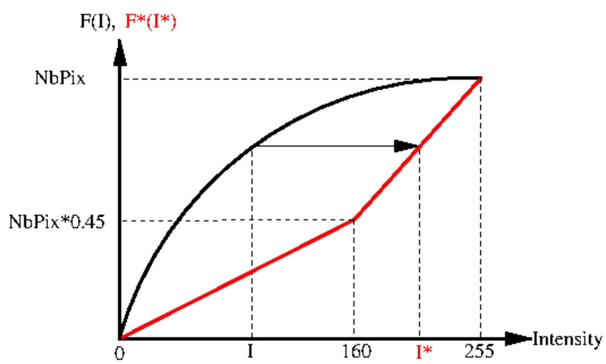


Figure 1: Histogram transformation, F is the cumulative distribution function of intensities I for the input image and F^* (in red) the desired cumulative distribution after transformation.

1. Explain the histogram equalization and the link with the proposed transformation.
2. Derive the algorithm for the proposed histogram transformation with the numbers shown in Figure (1), i.e. for an input intensity I what is the transformed intensity I^*.
3. How could we extend this algorithm to transform color images instead of intensity images while preserving colors ?

4 Color Compression (5 points)

We would like to compress color image files by reducing the size of the pixel color information. This information is typically encoded with 8 bits per channel and over 3 channels (i.e. RGB). We would like to use K-means for that purpose.

1. Explain shortly the K-means method and what it optimizes.

2. We would like to encode pixel color information over 6bits, for the full set of colors in the input image, and such that 3bits encode the R channel, 2 bits encode the G channel and 1 bit encodes the B channel :
   (a) How many colors can a compressed images have ?
   (b) Depict the corresponding k-means algorithm;
   (c) How could we store the corresponding transformed image into a file ?
3. Having different encoding sizes for the R,G and B channels is not very interesting in practice. Are there other file formats for which it would be more interesting ?