Image Formation

Introduction to Visual Computing

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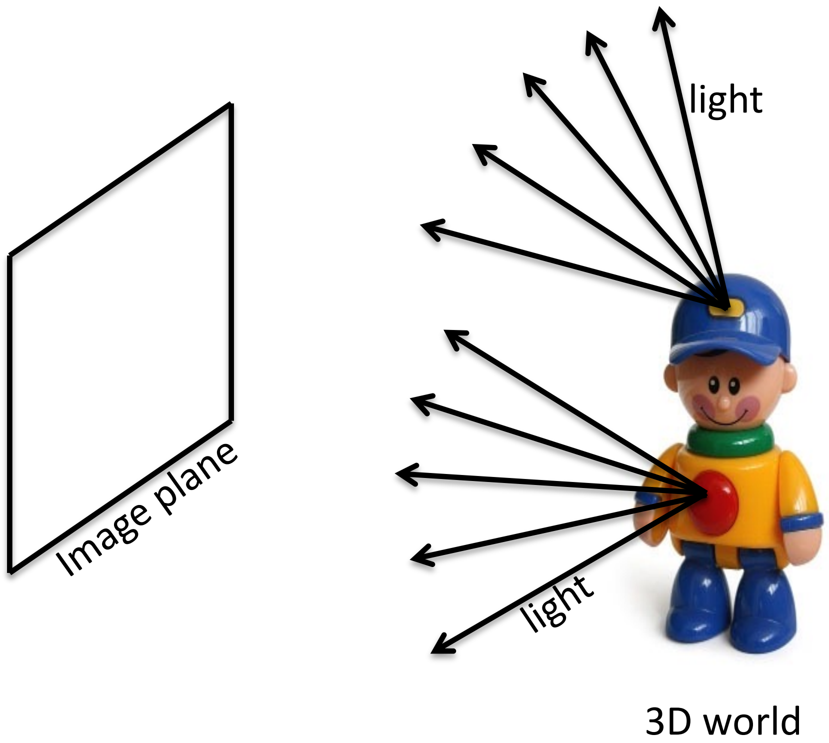
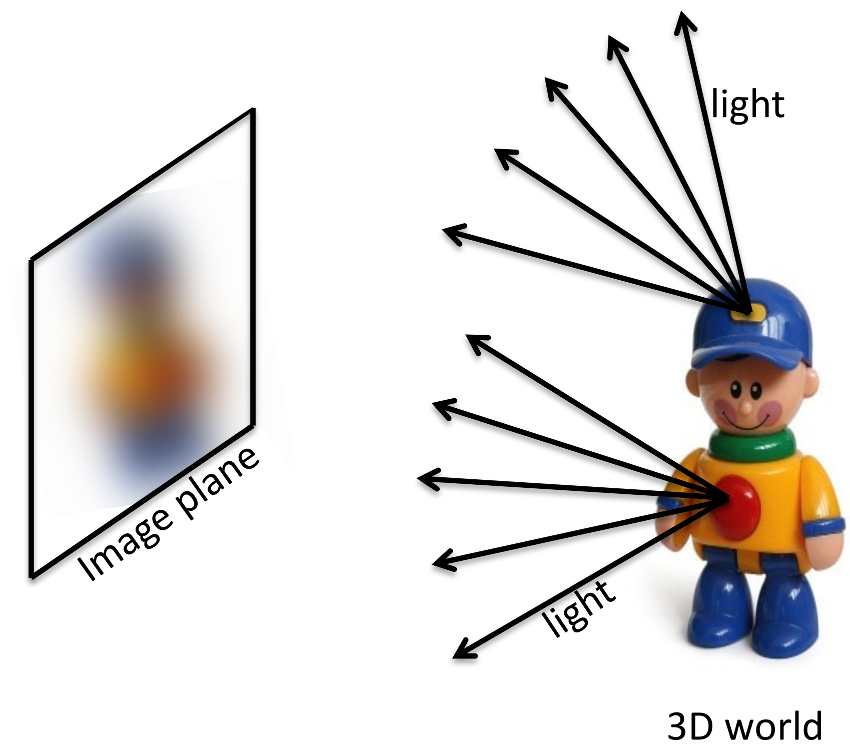
A real image is a 2D mapping of a 3D world scene

From 3D world to 2D images



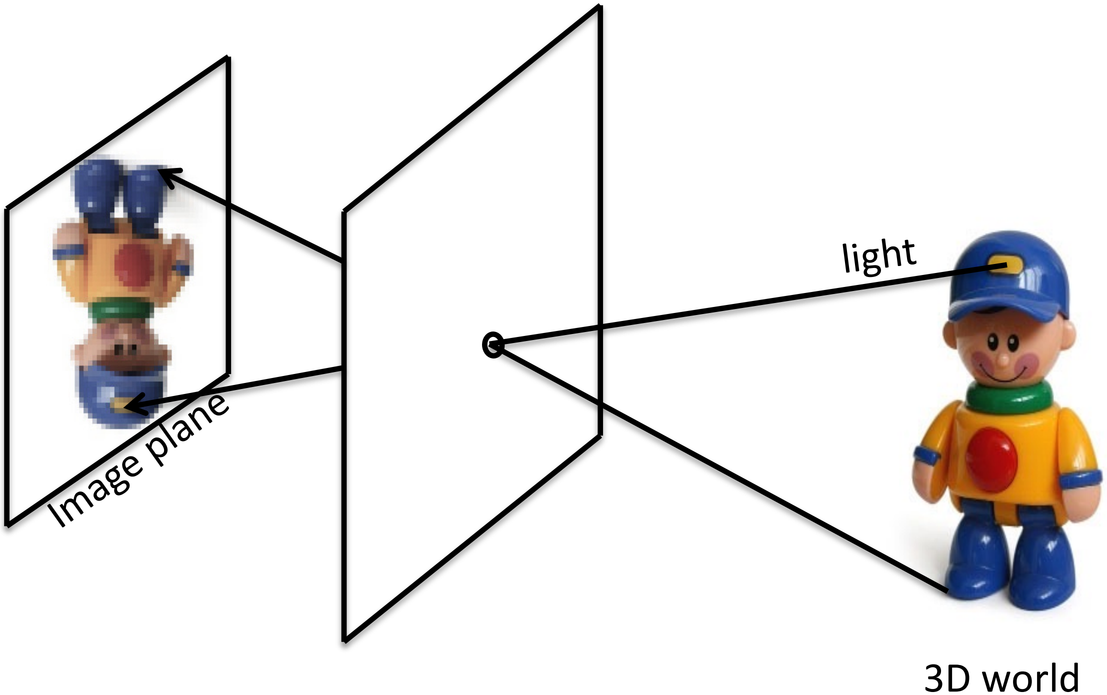
Light rays are emitted from the object in all directions

From 3D world to 2D images



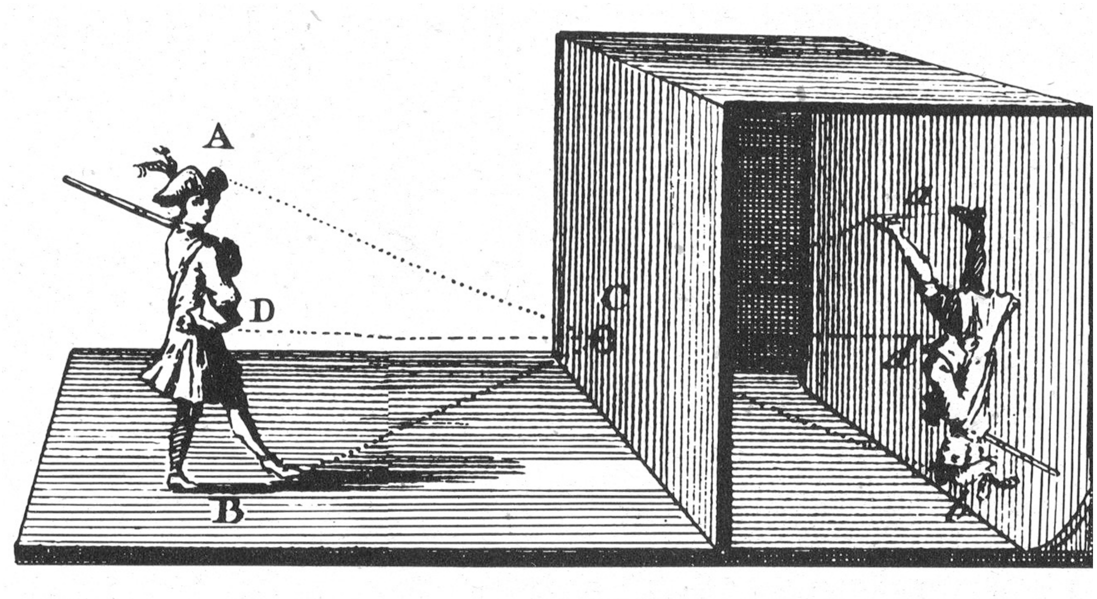
A point in the image plane receives rays from numerous 3D points

From 3D world to 2D images



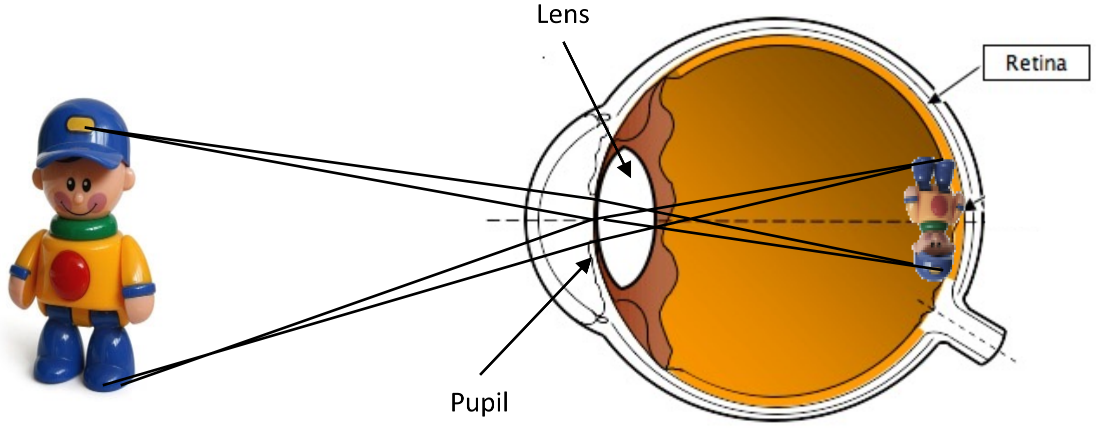
Pinhole photography: limit rays passing through using a small hole which size is called the aperture.

From 3D world to 2D images



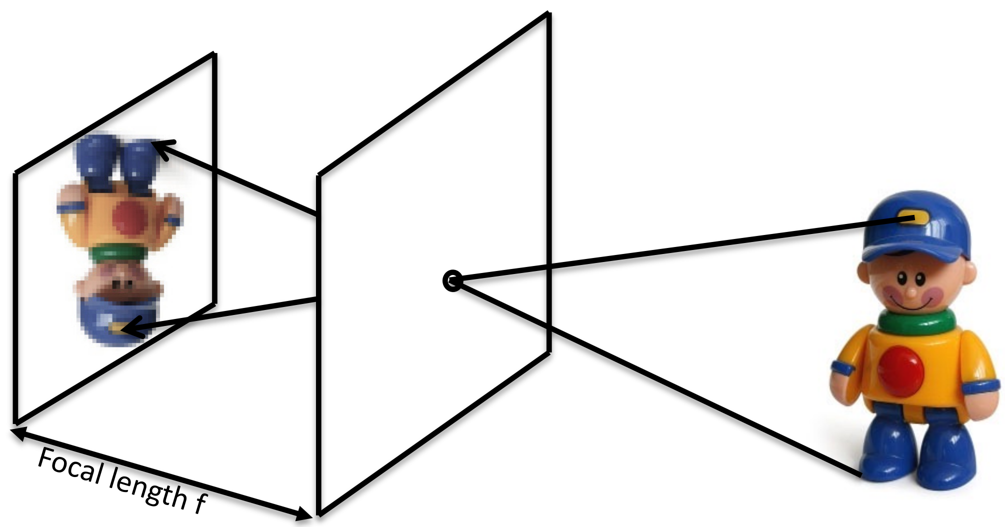
Principle known for centuries and called *camera obscura (dark room)*

From 3D world to 2D images



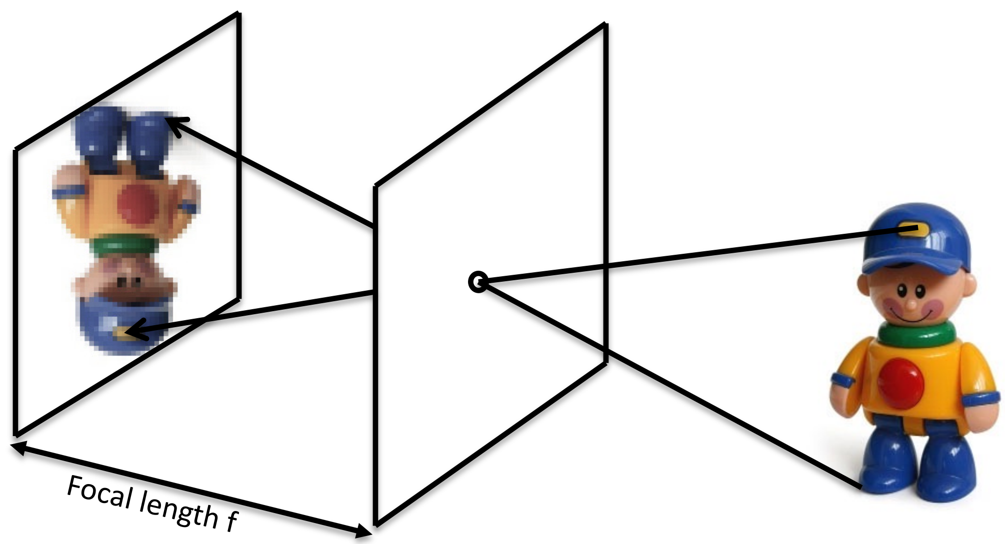
A similar principle holds for the eye where the pupil is the aperture and the retina the (photosensitive) image surface.

From 3D world to 2D images



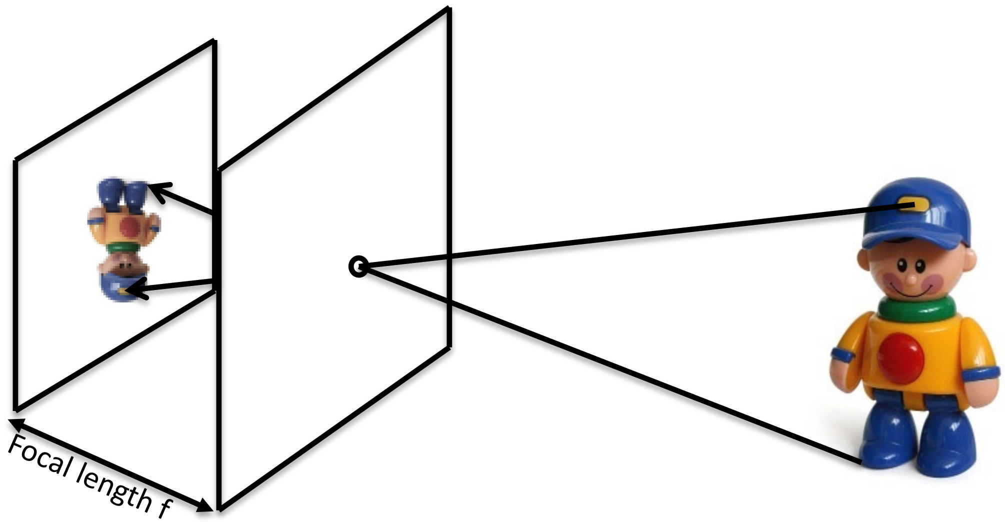
Key aspects: the distance between the hole and the image plane is the focal length.

From 3D world to 2D images



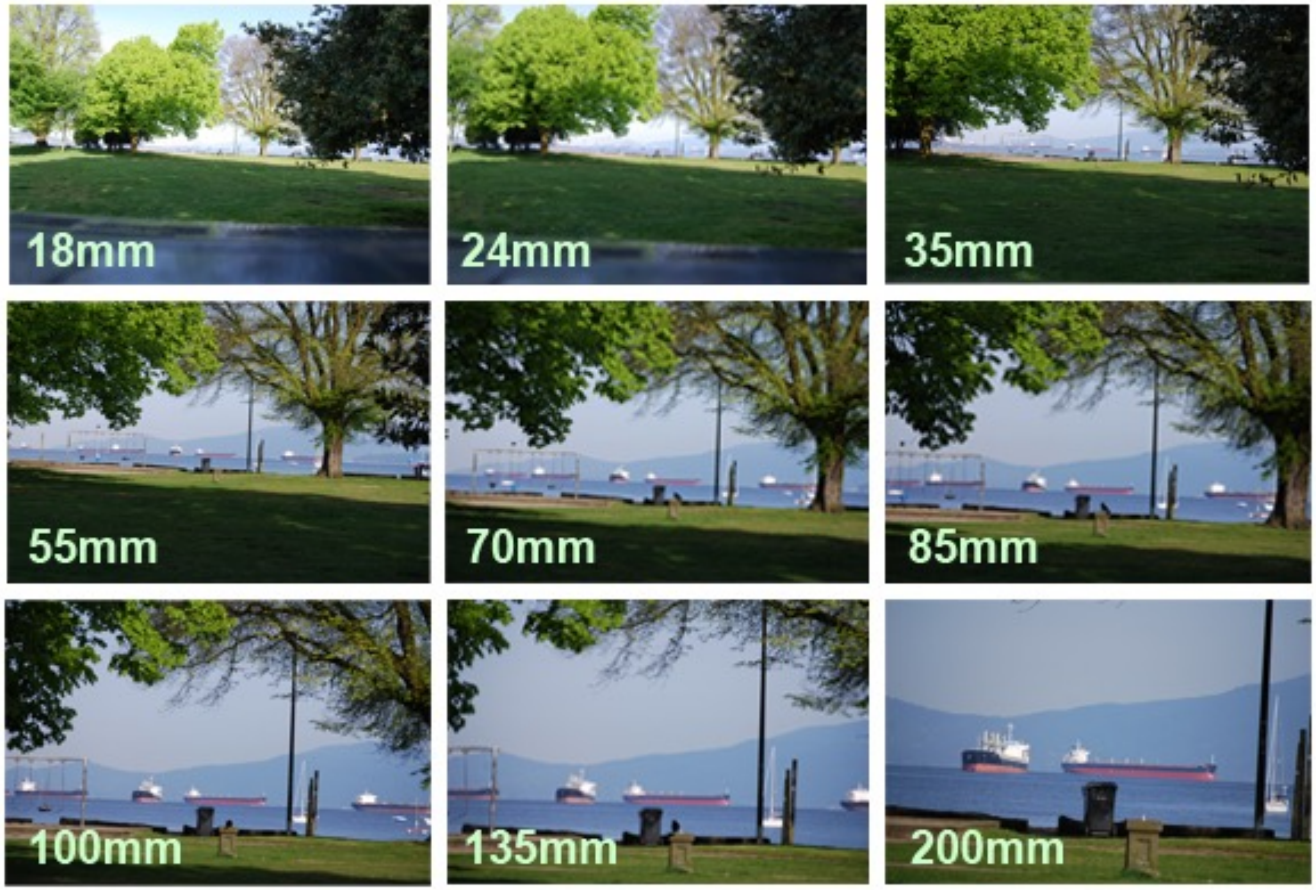
Key aspects: longer focal lenghts increase the object size and favor long distance shooting (zoom).

From 3D world to 2D images



Key aspects: shorter focal lengths favor short distance shooting.

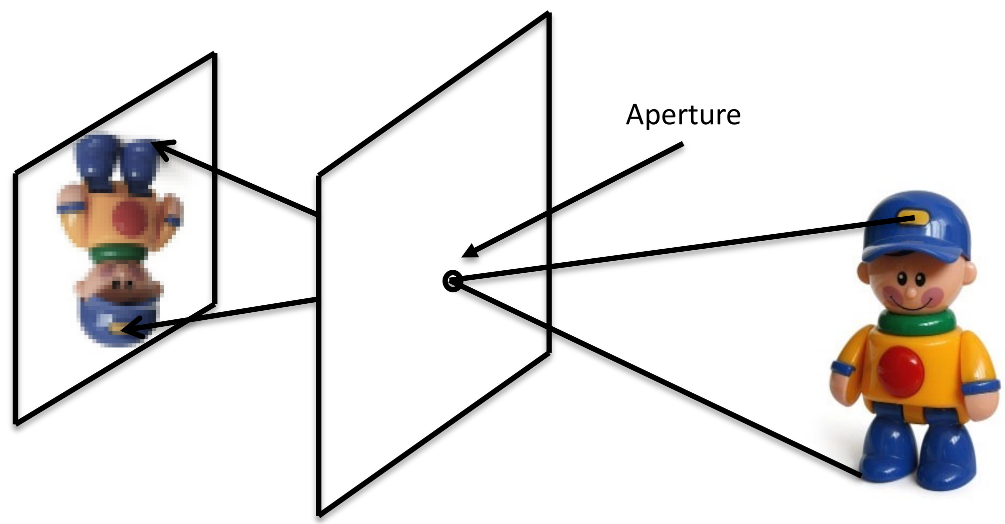
From 3D world to 2D images



http://info.photomodeler.com/

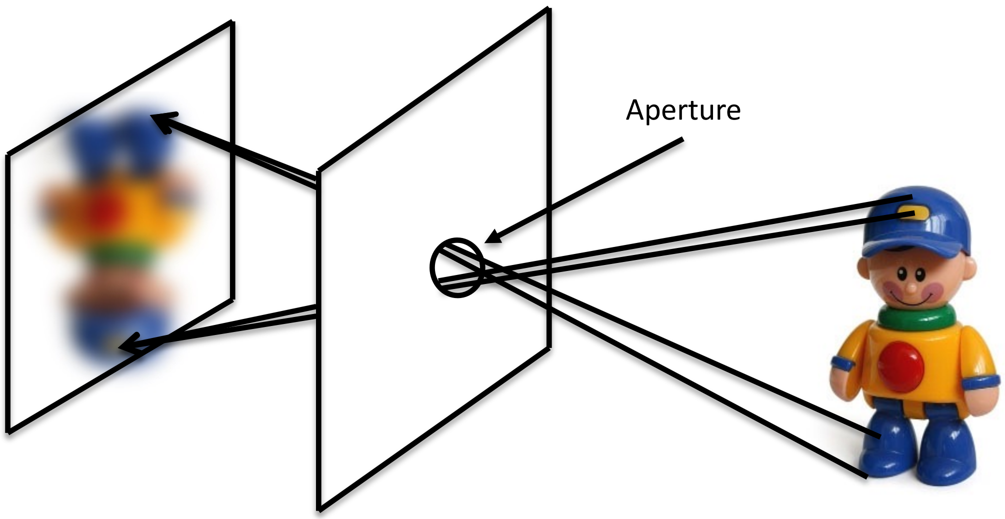
Examples of camera focal lengths (usually between 5 and 500mm).

From 3D world to 2D images



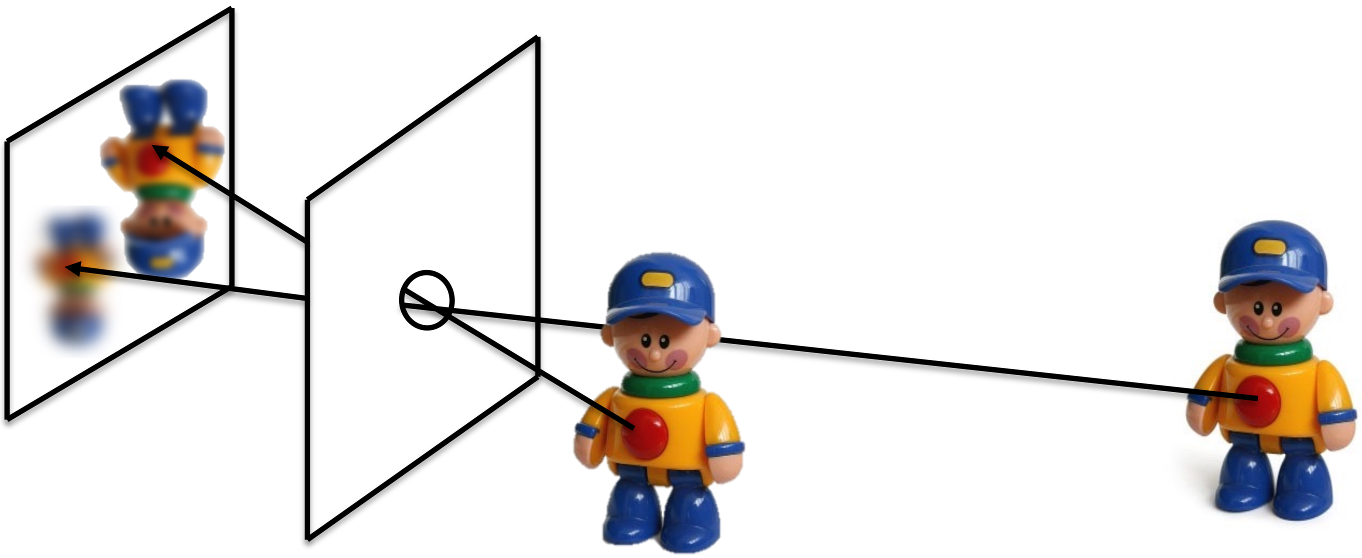
Key aspects: the **aperture** size.

From 3D world to 2D images



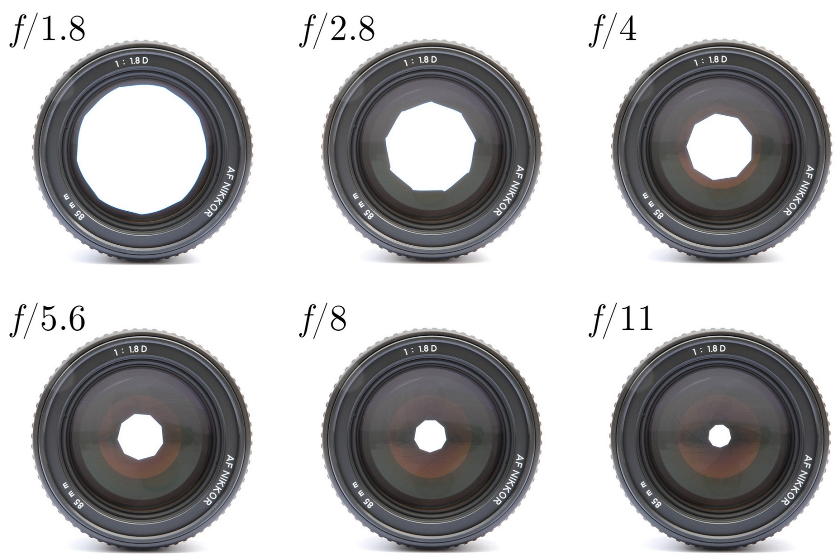
Key aspects: larger apertures increase the amount of light (brighter images) but also the blur.

From 3D world to 2D images



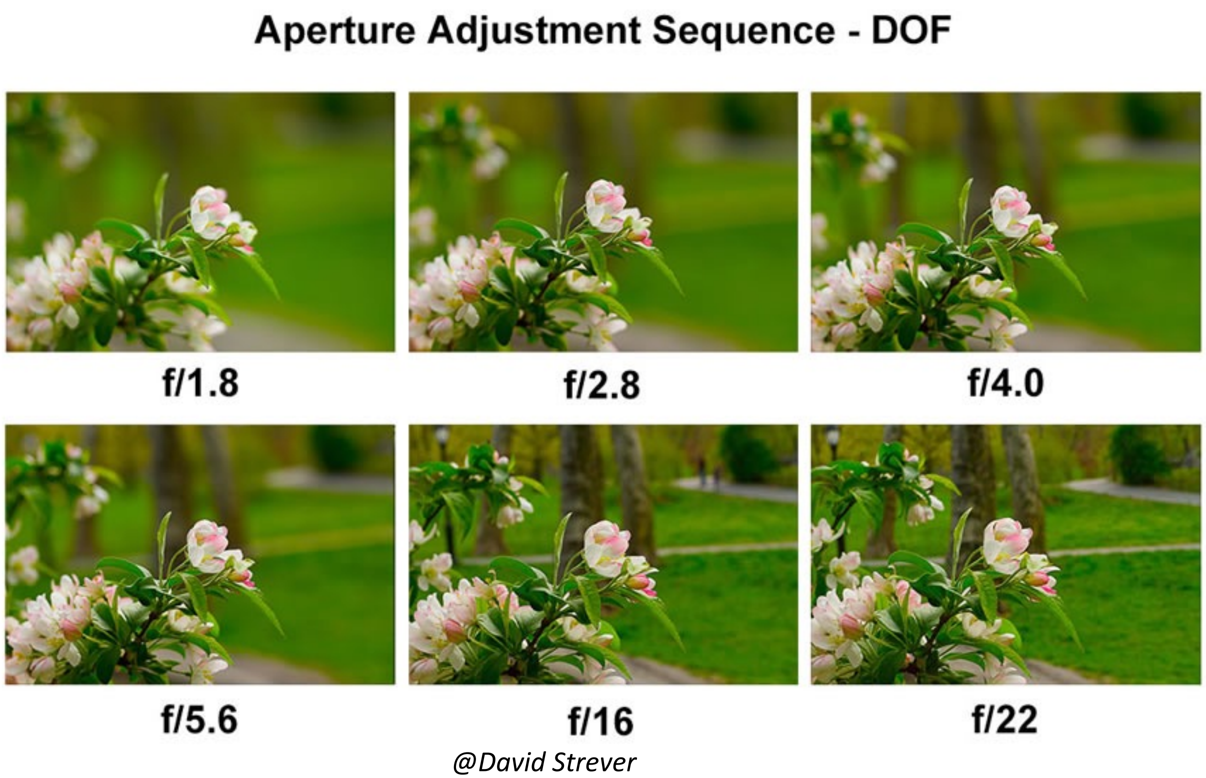
Key aspects: the blur increases with the distance to the image plane.

From 3D world to 2D images



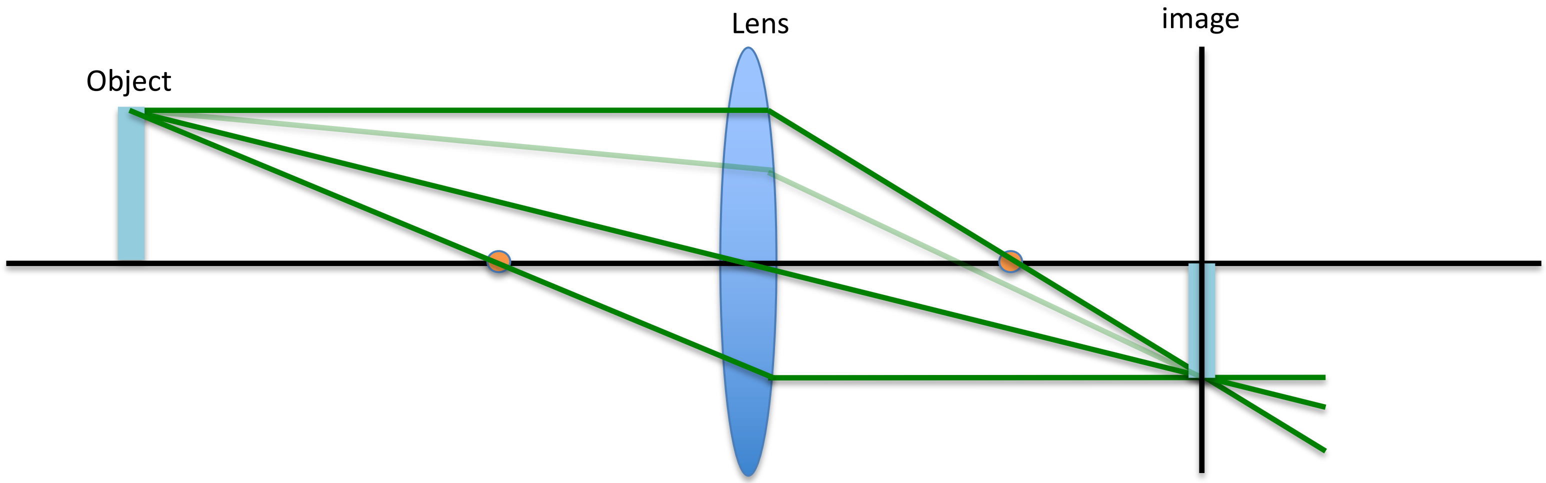
Aperture sizes in practice: on a camera a diaphragm controls the size.

From 3D world to 2D images



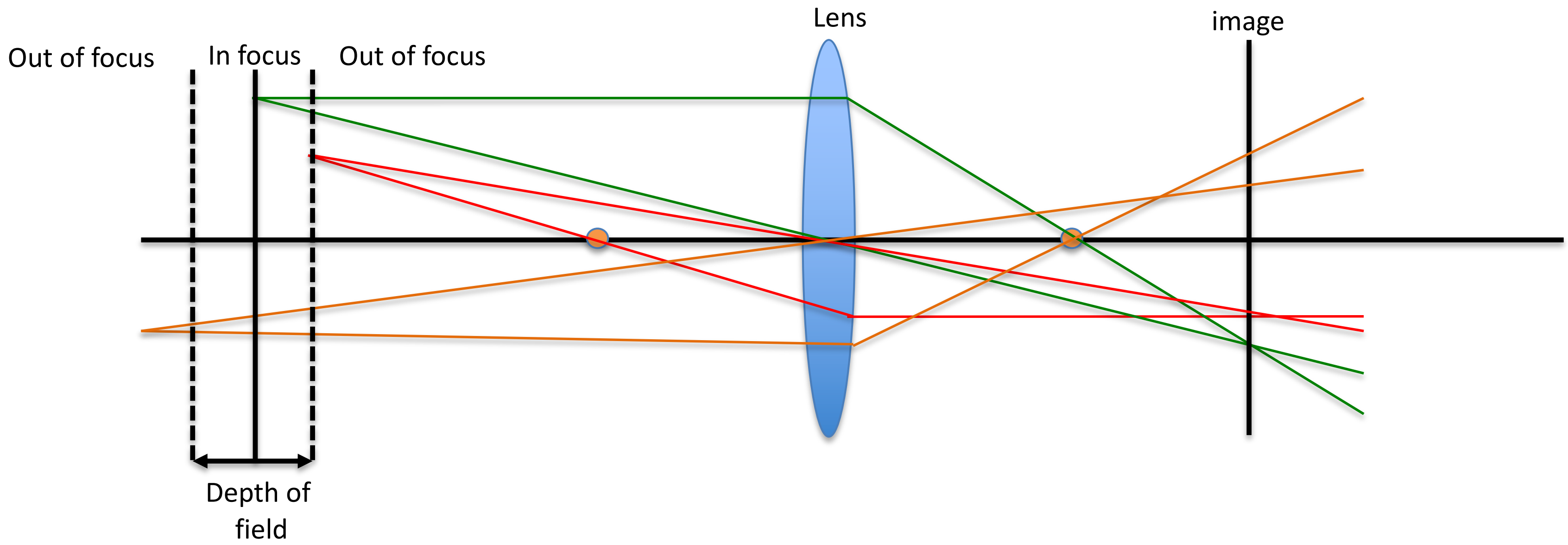
Aperture sizes in practice: examples.

From 3D world to 2D images



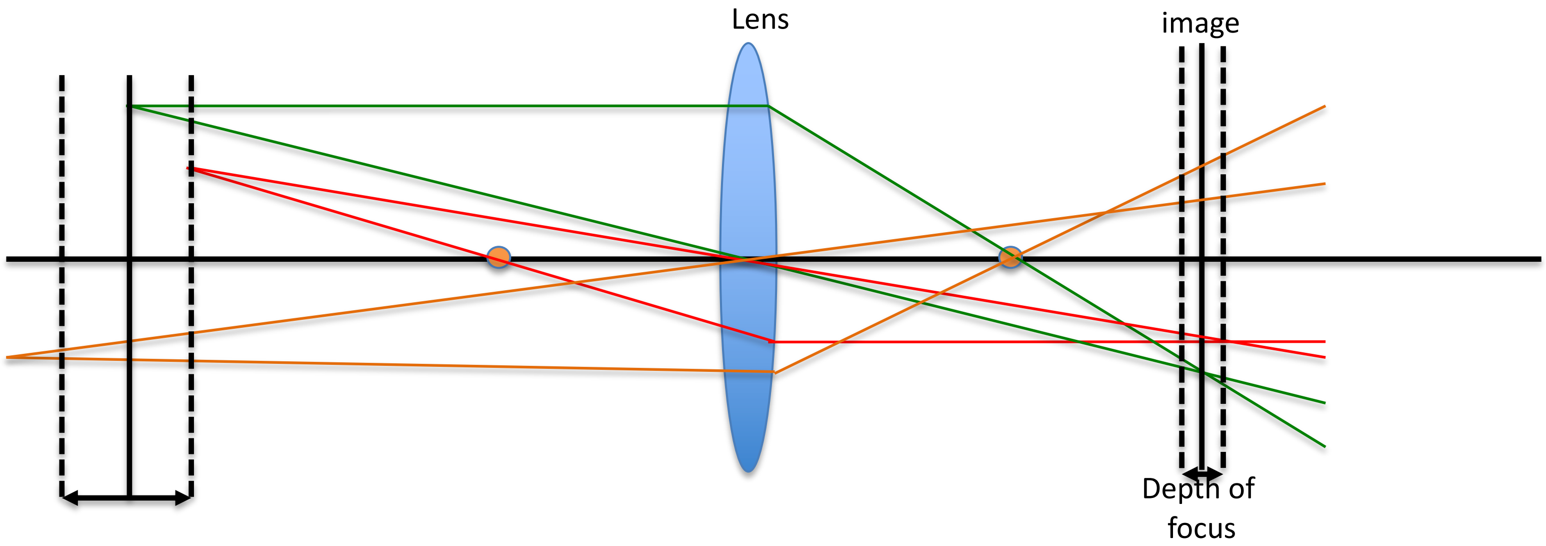
In practice, **lenses** are used to concentrate the light and compensate for the blur with pinholes and apertures.

From 3D world to 2D images

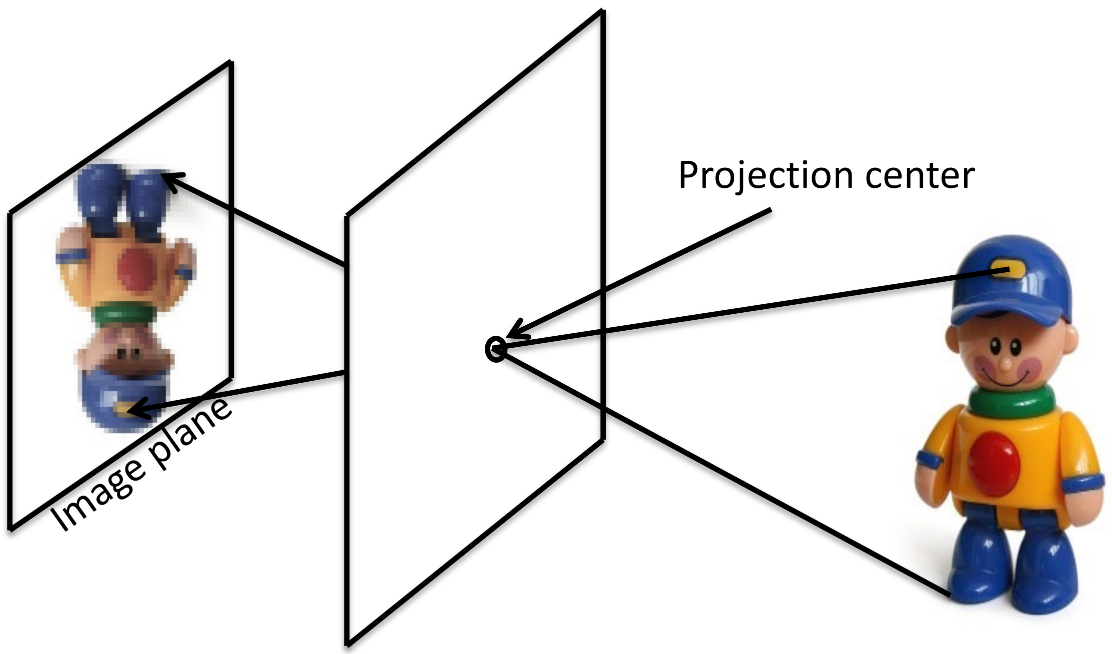


Lenses define in focus and out of focus area. The depth of field is then the range of distances where object are in focus, i.e. not fuzzy in the image.

From 3D world to 2D images

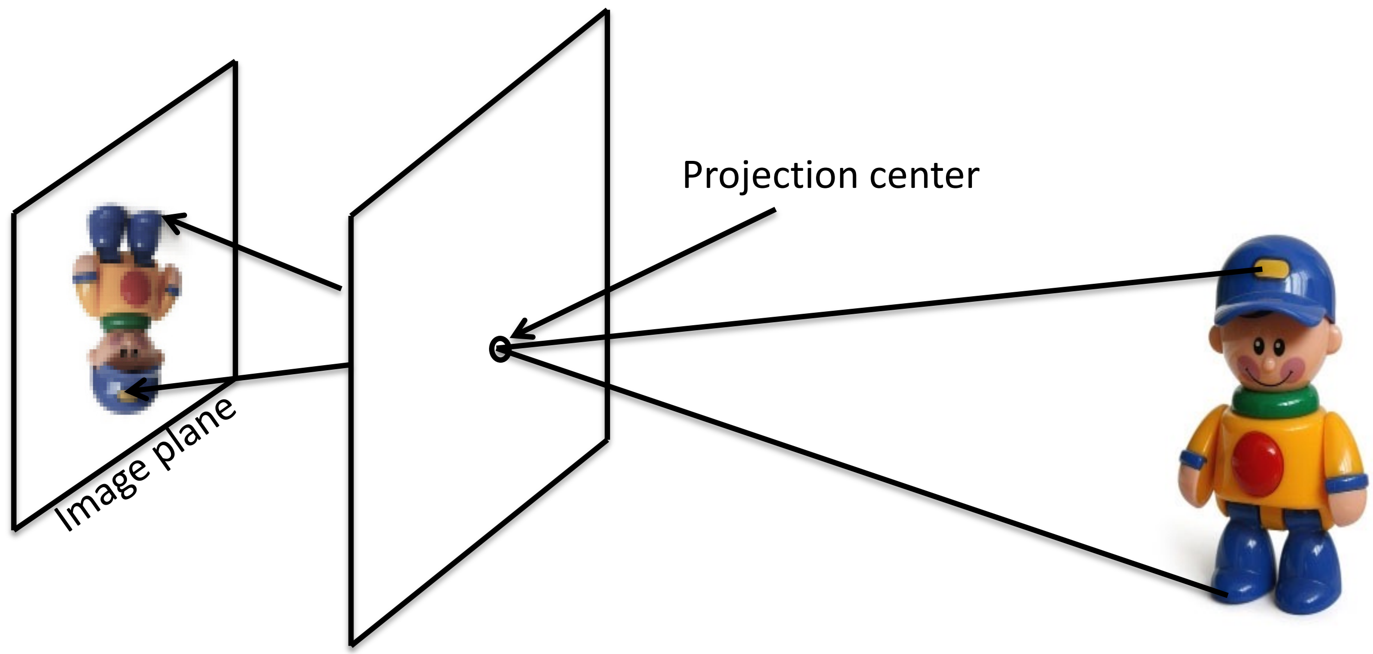


From 3D world to 2D images



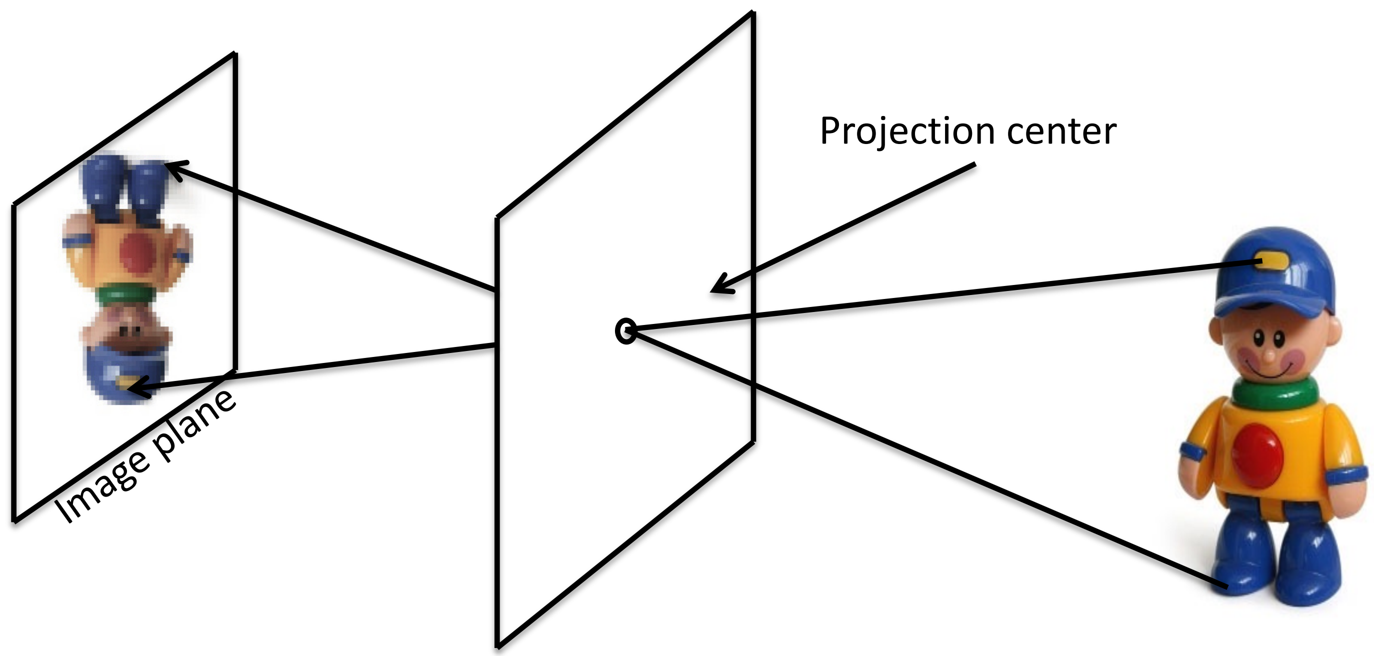
Perspective: going back to our pinhole model, all rays go through the pinhole which is the center of a perspective projection.

From 3D world to 2D images



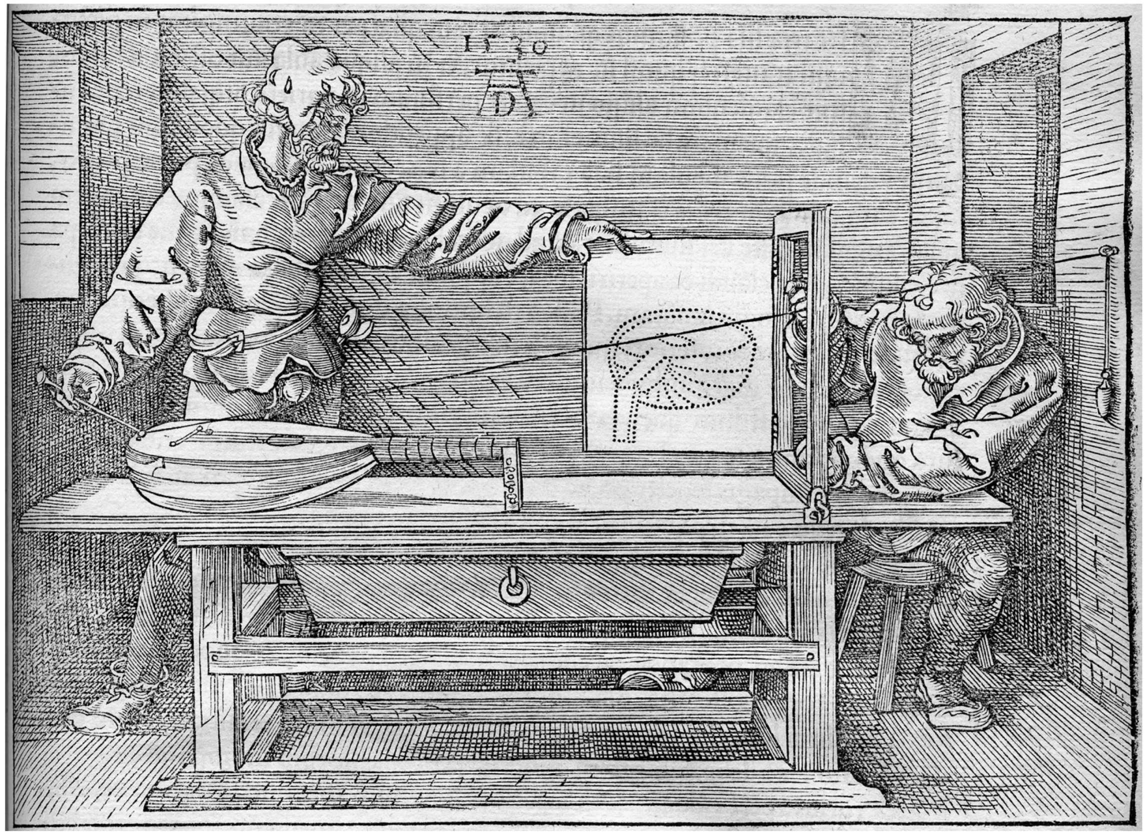
Perspective: objects that are further appear smaller in the image.

From 3D world to 2D images



Perspective: increasing the focal length (the perspective projection parameter) reduces the perspective effect.

From 3D world to 2D images



The Painter's Manual by Albrecht Dürer in 1538

Perspective: known, studied and used by Renaissance painters.

From 3D world to 2D images

La cène, Leonardo Da Vinci

Perspective: objects further appear smaller, hence parallel lines are not necessarily parallel.

From 3D world to 2D images

La cène, Leonardo Da Vinci

3D parallel lines intersect at vanishing points in the image

From 3D world to 2D images

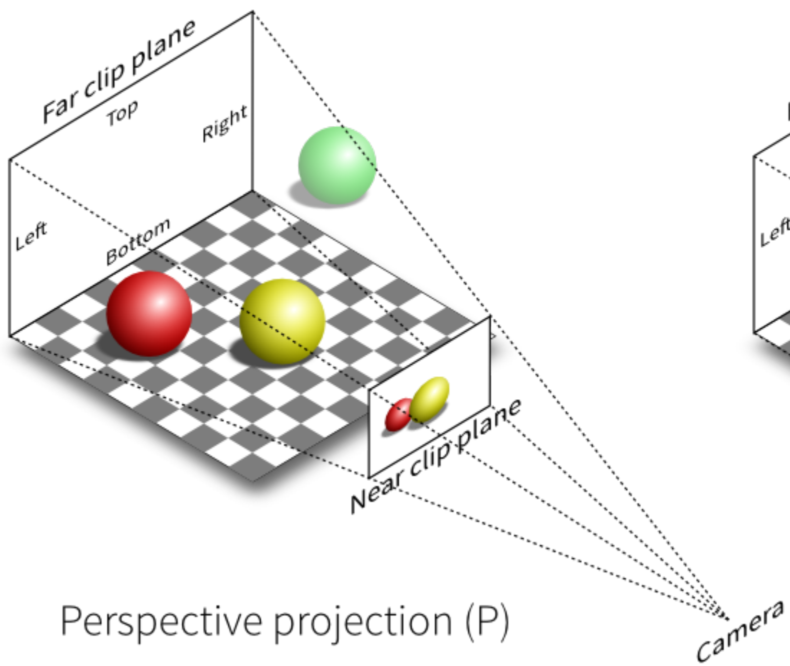
expertphotography.com

Different perspectives with increasing focal lengths and distances to the objects

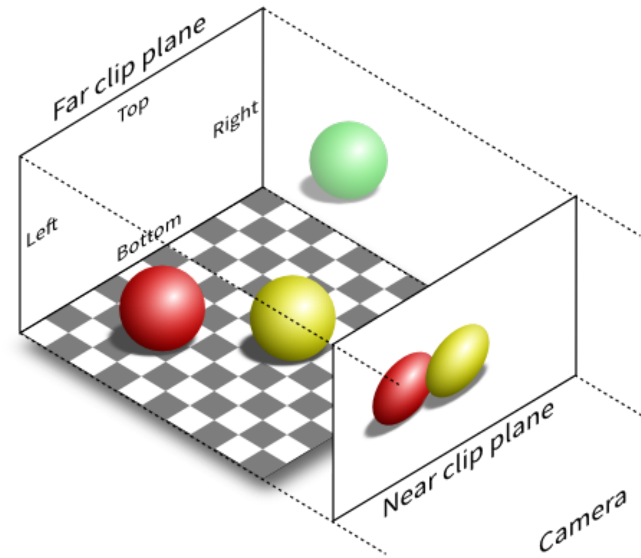
From 3D world to 2D images

Perspective illusion

From 3D world to 2D images



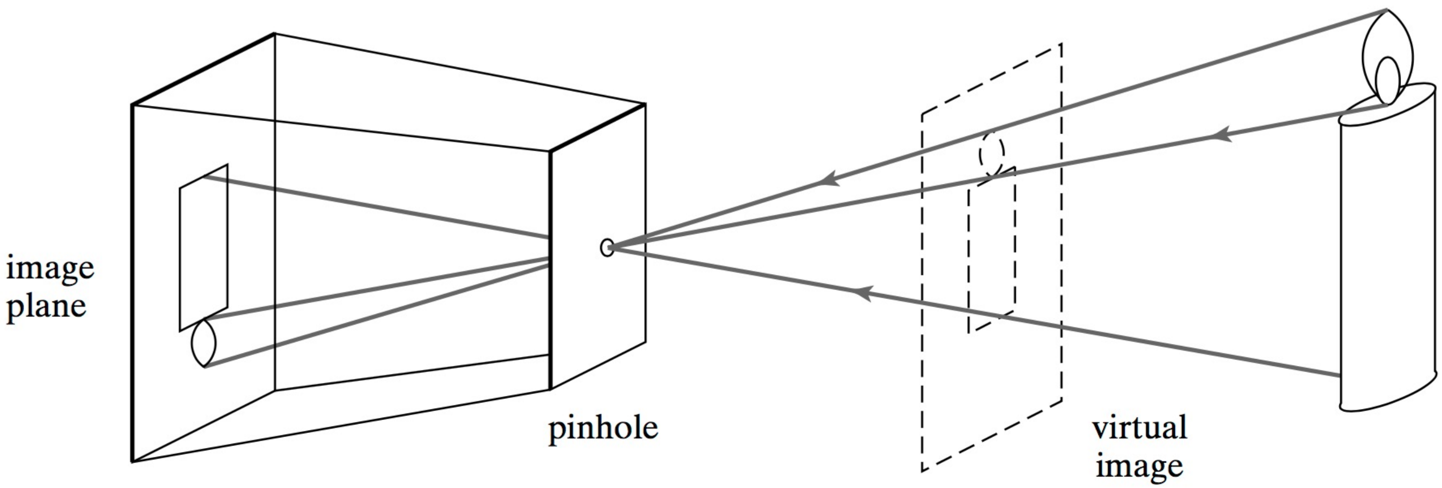
Perspective projection (P)



Orthographic projection (O)

In visual computing, simplified pinhole projections are generally considered, e.g. the above examples are image synthesis models with clipping volumes

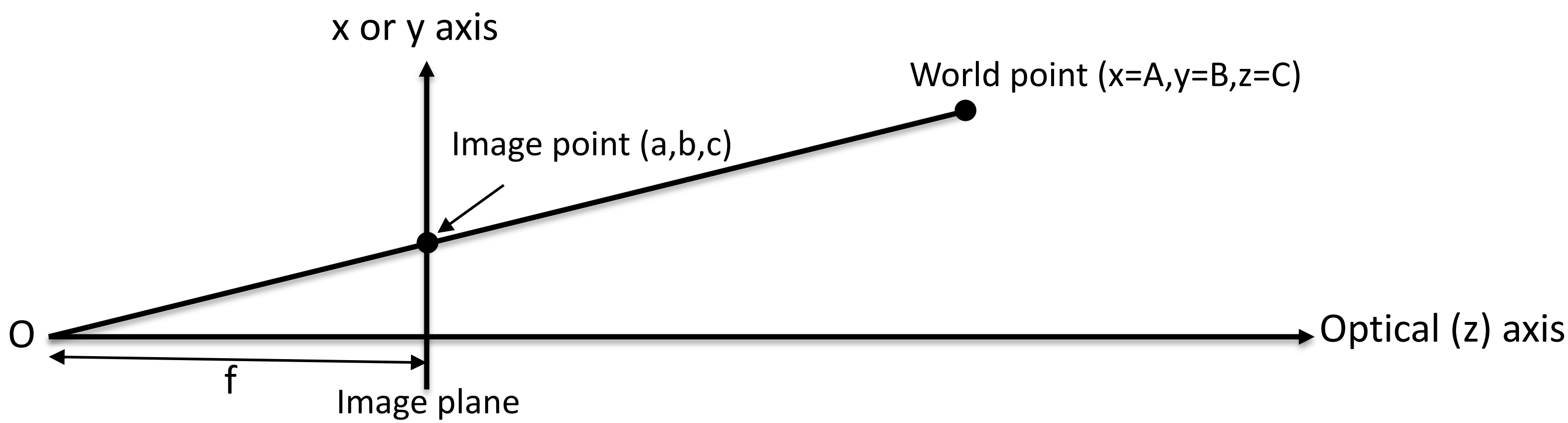
A bit of geometry



Forsyth & Ponce, computer vision book

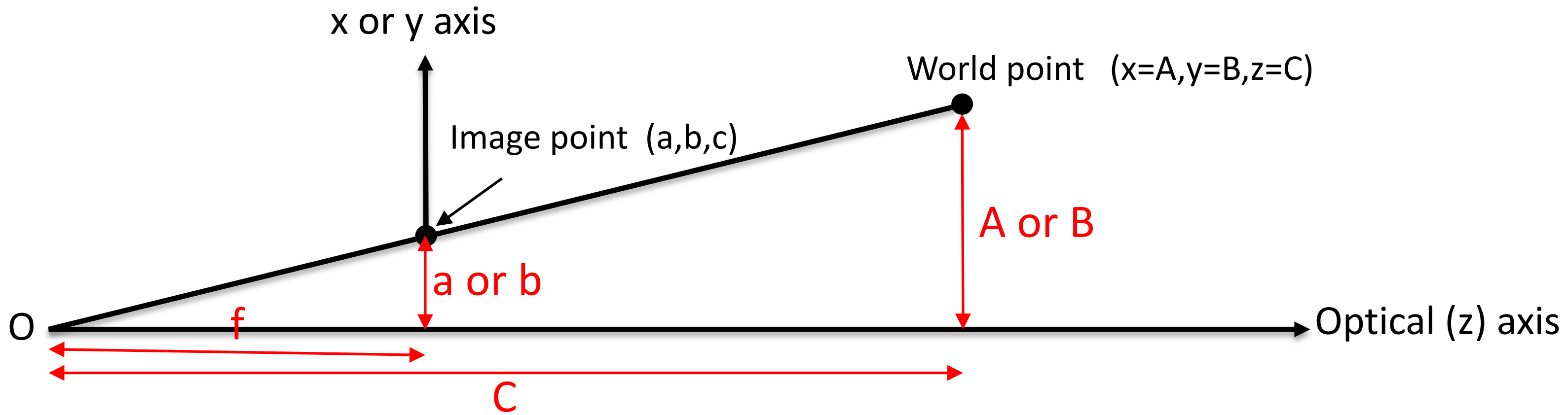
A simplified model: all the viewing rays go through a single point, the projection centre. The model will consider the virtual image, instead of the real image, to represent the 3D-2D projection.

A bit of geometry

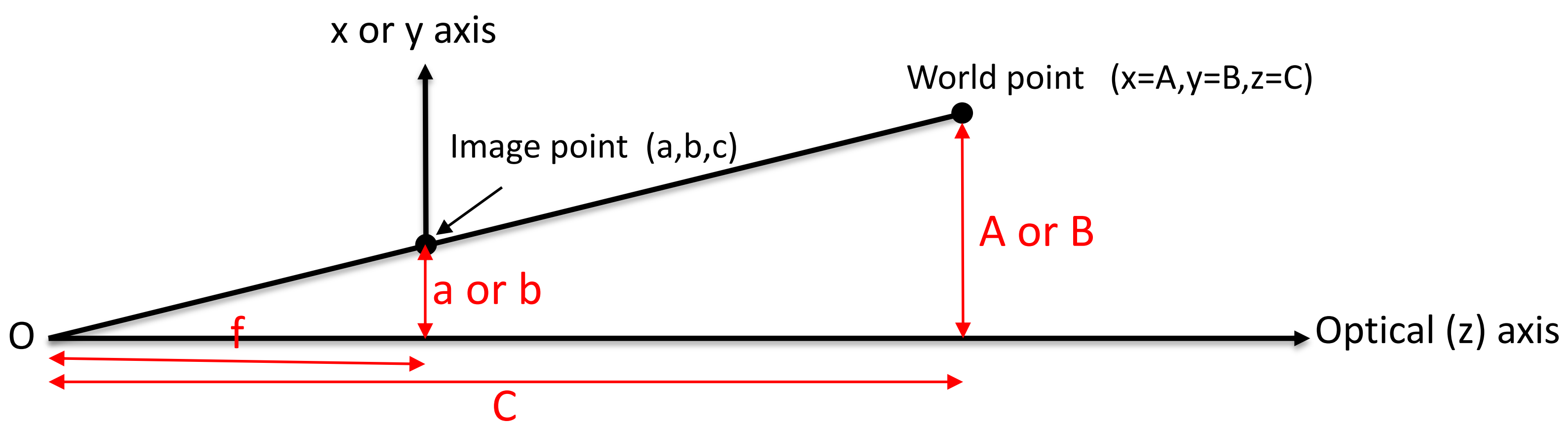


A simplified model: O is the projection centre, the optical axis the ray going through O and perpendicular to the image plane, f the focal length.
For a given point in 3D with coordinates (A,B,C) how do we determine its image point (a,b,c) ?

A bit of geometry



A bit of geometry



Using thales: A/a = C/f and B/b=C/f

Thus: a=f A/C, b= f B/C, c=f.

The next step (computational wise) is to represent perspective projection as a linear transformation in a matrix form (in order to ease the manipulation and combinations with other transformations).

A bit of geometry

Homogenous coordinates (borrowed from projective geometry): a 3D point P is represented by 4 coordinates (x,y,z,w), at least one of which is non zero, defined up to a scalar factor:
(x,y,z,w) and (λx,λy,λz,λw), λ≠0, represent the same point.

We assume that (x/w,y/w,z/w) are the traditional Euclidean coordinates.

The benefit is that all the linear transformations, all transformations that preserve coincidence relationships (e.g. perspective projections) in 3D can be represented with matrices using the homogeneous coordinates:

A bit of geometry

Translation:

$$T = \begin{bmatrix} 1 & 0 & 0 & Tx \\ 0 & 1 & 0 & Ty \\ 0 & 0 & 1 & Tz \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation:

$$R = \begin{bmatrix} & [\overline{R}] & & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \left\| \overline{R} \right\| = 1$$

Rigid transformation:

$$[x', y', z', 1]^t = \begin{bmatrix} & [\overline{R}] & & \begin{matrix} Tx \\ Ty \\ Tz \end{matrix} \\ 0 & 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} x \\ y \\ y \\ 1 \end{bmatrix}$$

A bit of geometry

3D world point

Perspective projection : $$\begin{bmatrix} X & Y & Z & Z/f \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/f & 0 \end{bmatrix} . \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \cong \begin{bmatrix} Xf/Z & Yf/Z & f & 1 \end{bmatrix}$$

Projected point in the image plane

Perspective projection matrix

Perspective projection with the origin in the image plane :

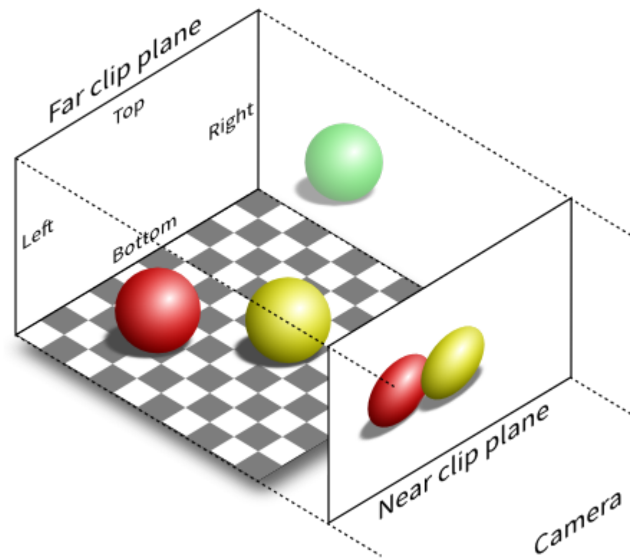
$$\begin{bmatrix} X & Y & 0 & 1+Z/f \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/f & 1 \end{bmatrix} . \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \cong \begin{bmatrix} Xf/(Z+f) & Yf/(Z+f) & 0 & 1 \end{bmatrix}$$

A bit of geometry

When f goes to infinity, we get the orthographic projection (i.e. parallel projection):

$$\begin{bmatrix} X & Y & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

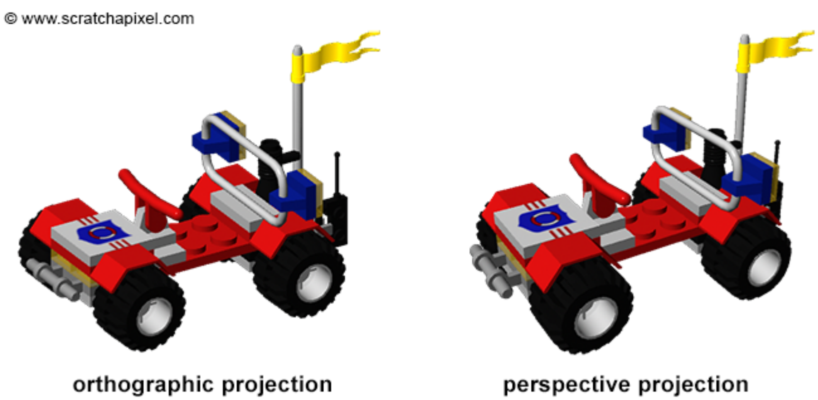
Projected point in the image plane



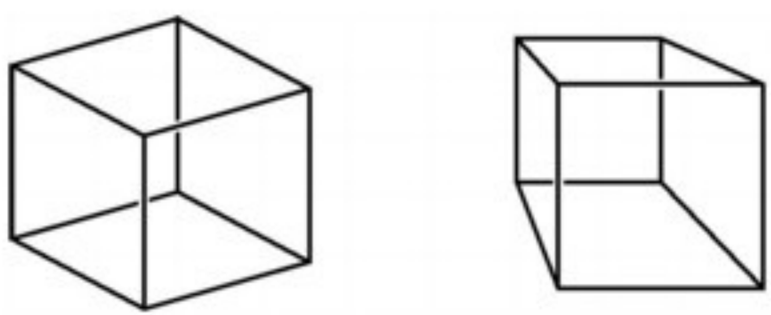
Orthographic projection (O)

Orthographic projection are good models when depth variations in the 3D scene are smaller than the scene size

A bit of geometry



Strong perspective with a clear vanishing point !



We do see in perspective (cube orientation) !

A bit of geometry

Vanishing points correspond to parallel line intersections, at infinity !

A bit of geometry

Assume d is the 3D direction of the parallel lines, then points on a line write: $\begin{bmatrix} x_0+\alpha d_x & y_0+\alpha d_y & z_0+\alpha dz & 1 \end{bmatrix}, \alpha \in \Re$ where $\begin{bmatrix} x_0 & y_0 & z_0 & 1 \end{bmatrix}$ is a point on the line and $\begin{bmatrix} d_x & d_y & d_z \end{bmatrix}$ correspond to the direction d.

In homogeneous coordinates we can multiply coordinates by scalar, thus: $\cong \begin{bmatrix} x_0/\alpha+d_x & y_0/\alpha+d_y & z_0/\alpha+dz & 1/\alpha \end{bmatrix}, \alpha \in \Re^*$ and when $\alpha \to \infty$ we get the 3D point with coordinates: $\begin{bmatrix} d_x & d_y & dz & 0 \end{bmatrix}$ which defines a vanishing point in the image.