All documents are allowed. The different sections below are independent. Answers should be **concise** *and* **justified**.

1 Projective Geometry (6 points)

1. Why is the Euclidean geometry not sufficient to model image formation ?
2. How many vanishing points the perspective projection of the edges of a cube define ?
3. Assume that a point P_i is linearly interpolated between two points P_1 and P_2 in 3D. Is the perspective projection of P_i the same linear interpolation between the projection of P_1 and P_2 ? Same question with an orthographic projection ?
4. What are the homogeneous coordinates of the line of $\mathcal{P}^2$ going through the points with homogeneous coordinates $(1, 0, 0)$ and $(0, 1, 0)$ respectively ?

2 Image Mosaics (4 points)

Assume that a camera acquires images while rotating about its optical center and assume further that the intrinsic parameter matrix K is the identity matrix.

1. Is the transformation between 2 such images projective or affine ?
2. Do we need 4 or 3 pairs of corresponding points to estimate this transformation ?
3. We want to build a cylindrical panorama using several images of that camera, detail the different steps of an algorithm for doing so.

3 Perspective Projection (6 points)

Consider a perspective projection with focal length f:

1. In such a projection why do objects further away appear smaller in the image ?
2. Given an object (perspectively) projected in an image how should I modify the focal length of the projection so that the size of the object in the image is divided by 2 ?
3. Assume that two spheres S_1 and S_2 of radius R and $2R$ are aligned along the optical axis (i.e. their centres lie on the optical axis) at distances D_1 and $D_2 \geq D_1$ from the projection center respectively.
   (a) Show that we observe two nested discs.
   (b) Assume D_1 fixed, above which distance D_2 will S_2 be fully occluded by S_1 ?
   (c) Assume the distance between the two spheres to be fixed, i.e. $D_2 - D_1$ is constant, at which distance D_1 will S_2 and S_1 project onto the same disc ?

4 3D Modeling (4 points)

1. What is the photoconsistency criterion ?
2. Assume that a textured 3D model is available. How can we perceive this model in 3D using a 2D device such as a mobile phone.
3. q1 and q2 are two image observations of a 3D point Q. Due to the noise, these points do not correspond to the exact projection of Q. As a result, the viewing lines of q1 and q2 do not intersect in 3D.
   (a) A good geometric approximation for Q is the point closest to both viewing lines. How can we estimate that approximation ?
   (b) Is there a closed form solution for it ?