All documents are allowed. The different sections below are independent. Answers should be **concise** *and* **justified**.

1 Projective Geometry (5 points)

1. What is the set of points of the projective plane $\mathcal{P}^2$ that are not present in the affine plane $\mathcal{A}^2$ of points with homogeneous coordinates $(x, y, 1)$? Can this set be defined with homogeneos coordinates ?
2. What are the homogeneous coordinates of the point of $\mathcal{P}^2$ intersection of the lines with homogeneous coordinates $(0, 0, 1)$ and $(0, 1, 0)$ respectively ?
3. Does this point belong to the line at infinity associated to $\mathcal{A}^2$?
4. Assume that C is the centroid of a set of n 2D points $\{P_i\}_{i\in[1..n]}$ in the affine space $\mathcal{A}^2$, i.e. C is the mean 2D position of $\{P_i\}_{i\in[1..n]}$. Is this centroid preserved by an affine transformation of the plane ? by a projective transformation of the plane ?

2 Plane Projection (5 points)

The perspective projection of the point P with homogeneous coordinates $(x, y, z, 1)$ onto the image point with coordinates (u, v) can be modeled with:

$$(wu, wv, w)^t \sim K[R \;\; -Rt](x, y, z, 1)^t, \tag{1}$$

where K is the intrinsic parameter matrix:

$$K = \begin{bmatrix} k_u f & 0 & u_0 \\ 0 & k_v f & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

R a rotation matrix in $\mathbb{R}^3$ and t the 3×1 position vector of the camera center in the world coordinate frame.

1. We consider points in the plane with equation $z = 0$, what kind of transformation becomes the above projection (1) with such points ?
2. Assume that the image plane is parallel to the plane with equation $z = 0$.
   (a) What kind of transformation is the above projection (1) with the points in the plane $z = 0$?
   (b) What is the minimum number of point correspondences required to estimate the transformation and how can we compute it given this number of correspondences ?
   (c) Consider two lines in the plane $z = 0$ that are parallel to the x axis. Where does the projections of those two lines intersect in the image plane ?

2.1 Image Mosaics (4 points)

Assume that a camera acquires images while rotating about its optical center and consider the case where the camera takes **two** images. Between the two images, it carries out a rotation about the Y axis:

$$\mathsf{R} = \begin{pmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{pmatrix}$$

As for the intrinsic parameters of the camera, we suppose that they correspond to a simplified calibration matrix whose only unknown is the focal length α:

$$\mathsf{K} = \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

It is known that there exists a projective transformation (homography) that links the two images. The goal of this question is to estimate the transformation from a single point correspondence.

1. Write down the homography H in terms of the two unknowns, the rotation angle β and the focal length α.
2. Develop a method (formulas) for computing these two unknowns, from a single point match: $\mathbf{q_1}$ in the first image and $\mathbf{q_2}$ in the second one.

3 3D Modeling (6 points)

1. We consider the estimation of the visual hull associated to n 2D silhouettes:
   (a) Depict shortly an algorithm that uses a voxel grid of size d^3.
   (b) What is the theoretical maximum number of *inside silhouette* tests required ? Is there a theoretical minimum number of such tests ?
2. Depending on the number of viewpoints available we can perceive a scene in 2D or 3D using adapted displays. How can we perceive 3D with a mobile phone, a stereo screen or a head mounted display ? explain how they differ.
3. In order to build 3D models from images some methods assume a a prior model of the observed scene, e.g. human body models. What is the advantage of such a strategy ? its limitations ?
4. q1 and q2 are two image observations of a 3D point Q. Due to the noise, these points do not correspond to the exact projection of Q. As a result, the viewing lines of q1 and q2 do not intersect in 3D.
   (a) Describe an approach to compute, along the viewing line of q1, the depth λ_1 of the 3D point on this line that is closest to the viewing line of q2.
   (b) Is there a closed form solution for λ_1 or do we need to solve a system of equations ?