All documents are allowed. The different sections below are independent. Answers should be concise and justified.

1 Projective Geometry (5 points)

1. What is the link beetween the affine plane $\mathcal{A}^2$ of points with homogeneous coordinates $(x, y, 1)$ and the projective plane $\mathcal{P}^2$?
2. What are the homogeneous coordinates of the line of $\mathcal{P}^2$ going through the points with homogeneous coordinates $(0, 0, 1)$ and $(0, 1, 0)$ respectively ?
3. Where does this line intersects the line at infinity associated to $\mathcal{A}^2$?
4. Assume that C is the centroid of a set of n points $\{P_i\}_{i\in[1..n]}$ in the affine space $\mathcal{A}^2$, i.e. C is the mean position of $\{P_i\}_{i\in[1..n]}$. Is this centroid preserved by an affine transformation of the plane ? by a projective transformation of the plane ?

2 Plane Projection (5 points)

The perspective projection of the point P with homogeneous coordinates $(x, y, z, 1)$ onto the image point with coordinates (u, v) can be modeled with:

$$(\alpha u, \alpha v, \alpha)^t \sim K[R \; -Rt](x, y, z, 1)^t, \tag{1}$$

where K is the intrinsic parameter matrix:

$$K = \begin{bmatrix} k_u f & 0 & u_0 \\ 0 & k_v f & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

R a rotation matrix in $\mathbb{R}^3$ and t the 3×1 position vector of the camera in the world coordinate frame.

1. We consider points in the plane with equation $z = 0$, what kind of transformation becomes the above projection (1) with such points ?
2. How many pairs of correspondences between world and image points are required to estimate this transformation ?
3. Assume that the projection is an orthographic projection, e.g. $-Rt = (0, 0, f)^t$ with a focal length f that becomes infinite. What kind of transformation is the above projection (1) with the points $z = 0$?

4. Assume that a sample of n pairs of matched point is required to estimate the projection and that a robust RANSAC strategy (see notes) is applied over $N > n$ pairs of matched points. Assume further that 90% of these matched pairs are inliers. How many samples must be picked in order to have 90% chance of getting a pure-inlier sample ?

3 Perspective Projection (5 points)

We consider in the following a perspective projection without image pixel transformation, i.e.:

$$K = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and with } R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } t = (0,0,0)^t.$$

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1. In this projection why do objects further away appear smaller in the image ?
2. Given an object (perspectively) projected in an image how should I modify the focal length of the projection so that the size of the object in the image is divided by 2 ?
3. Assume that a cube of edge dimension e is observed with a perspective camera at distance d (from the projection center to the cube center) and a focal length f. Assume further that the cube is aligned with the image plane so that its projection forms 2 nested squares. (a) What is the exact length difference between the 2 square edges in the image ? (b) How should we modify the projection so that the smaller square gets bigger in the image while keeping the bigger square dimension constant, i.e. reducing the perspective effect ?

4 3D Modeling (5 points)

1. Considering a point on a shape silhouette in an image, can we tell whether the corresponding point in 3D is a convex, concave or saddle point on the observed shape ?
2. An algorithm estimates the visual hull associated to n silhouettes using a voxel grid of size d^3. What is the theoretical maximum number of *inside silhouette* tests required ? Is there a theoretical minimum number of such tests ?
3. Depending on the number of viewpoints available we can perceive a scene in 2D or 3D using adapted displays. How can we perceive 3D with a mobile phone, a stereo screen or a head mounted display ? explain how they differ.
4. In a multi-view stereo reconstruction, we seek for points that are photoconsistent, what does it mean to be photoconsistent and what are the assumptions made in such reconstruction ?
5. A 4D model (hologram) is composed of shape, appearance and sometimes motion information, explain what are these information ?