Computer Vision

All documents are allowed. The different sections below are independent. Answers should be explained and concise.

1 Projective Geometry (4 points)

1. Why is the projective geometry useful in image analysis ?
2. Where is infinity in an affine space ? and in a projective space ?
3. Show that parallelism is preserved under affine transformations in P^2.
4. How many correspondences between pairs of points are required to estimate a similitude transformation in P^2 ?

2 Image Mosaics (4 points)

A camera acquires images while rotating around its projection center.

1. Show that the transformation between any 2 such images is a 3×3 homography.
2. How can we compute this homography given 2 images (detail your answer) ?
3. In order to build the mosaic, all images are projected onto the same planar domain, e.g. a plane or a cylinder. Sketch an algorithm for doing so.

3 Establishing correspondence of lines (4 points)

The epipolar geometry gives us a means to establish correspondence between points in two images: two points are correspondences if they lie on the respective epipolar lines.

Here, we examine the same scenario, but now for lines. Concretely, we consider two cameras, whose position, orientation, and intrinsic parameters are all known (see figure 1, right). We further consider

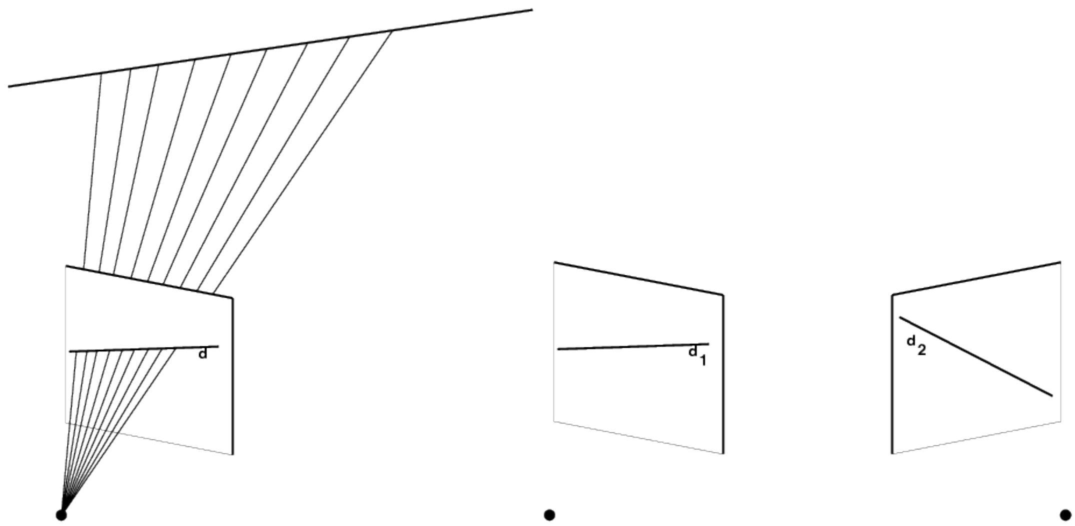

Figure 1: Left: Projection of a 3D line onto an image. Right: The scenario considered here: two cameras, with one line in each image.

one line $\mathbf{d_1}$ in the first image, and one line $\mathbf{d_2}$ in the second one. We assume that each of these lines is the projection of a line in 3D.

1. Is there a means to decide if the lines $\mathbf{d_1}$ and $\mathbf{d_2}$ are possible correspondences, i.e. if they can be projections of a single 3D line ? Try to answer using geometric arguments only, without giving formulas (they are too complicated).
2. Suppose now that we have 3 cameras and consider one line in each image. Is there a means to decide if all 3 lines are possible correspondences, i.e. if they correspond to a single 3D line ?

4 Homography (5 points)

Assume that points in a plane P are projected onto image I_1 and I_2 (see Figure 2) with the 3x4 projection matrices $M_1 = [KR_1\ KT_1]$ and $M_2 = [KR_2\ KT_2]$ respectively, where K is the 3x3 intrinsic parameter matrix seen during the lectures (upper triangular matrix); R_1, R_2 two 3x3 rotation matrices and T_1, T_2 two 3x1 translation vectors. Rotations and translations are expressed in the world coordinate frame. This coordinate frame originates in the plane P and is such that $z = 0$ for points in P.

1. Show that when the plane of image I_2 is parallel to P, i.e. when R_2 is the identity matrix, the transformation between P and I_2 is an affine transformation.
2. Show that the transformation between I_1 and I_2 is a homography of the plane.

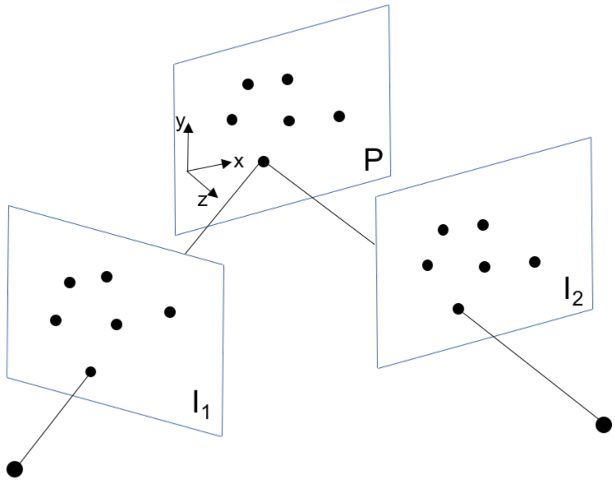

Figure 2: Homography

3. Assume that we are given 6 pairs of correspondences between I_1 and I_2 and that a robust RANSAC estimation approach is used to estimate the homography. How many outliers can be present in the matched pairs to still have more than 50% chance to correctly estimate the homography ?

5 3D Modeling (3 points)

1. Consider the torus shape. Can it be fully reconstructed from silhouette information ? how many silhouettes are required to this purpose ?

2. In computer vision, a reconstruction strategy consists in discretizing the 3D space into regular cubic cells, the voxels. Sketch an algorithm to estimate the visual hull with voxels given a set of silhouettes $\{S_i\}$.

3. Silhouette information do not fully describe shapes, what additional information is usually considered when reconstructing using images ? how is such information used ?