Computer Vision

All documents are allowed. The different sections below are independent. Answers should be explained and concise.

1 Projective Geometry (4 points)

1. What is the difference between a projective space and an affine space ?
2. Can two lines be parallel in a purely projective space ?
3. Show that the intersection of two lines is preserved under projective transformations.
4. How many correspondences between pairs of points are required to estimate an affine transformation in P^n ?

2 Epipolar geometry (5 points)

The essential matrix between two images is given by (see notes):

$$E \sim R_2[t_1 - t_2]_\times R_1$$

1. This allows to compute an epipolar line in image 2, from a point in image 1. What is the essential matrix for the other direction (going from image 2 to image 1) ?
2. Assume that a sample of 5 pairs of matched point is required to estimate E and that a robust RANSAC strategy (see notes) is applied over N > 5 pairs of matched points. Assume further that 20% of these matched pairs are outliers. How many samples must be picked in order to have 90% chance of getting a pure-inlier sample ?
3. Assume the motion between the cameras is a pure translation along the image horizontal direction. Where are the epipoles in this case and what are the pipolar lines ?
4. Assume the motion is a pure rotation, do we still have epipolar constraints ?
5. We consider now 3 images with known epipolar geometries. Given a point in one image how can we find its corresponding points in the other images using the epipolar geometries ?

Figure 1: Perspective correction

3 Perspective Correction (5 points)

Several image processing tools allow to correct distortion effects due to perspective. The example in Fig.1 shows such a correction where the input image on the left has been distorted on the right.

1. What kind of transformation is applied to the left image to get the right image ? would an affine transformation work ? please justify your answer.
2. The observed scene is more or less planar, what is therefore the transformation between the observed plane and the image plane on the right ? can it be an affine transformation ?
3. In order to estimate the correction a first strategy is to ask the user to pick points in the left image and to give their new positions in the right image. How many such points in the left image must be picked for the correction estimation ? given these points and their new positions how do we estimate the correction ?
4. Assume we are given the intrinsic parameters of the camera, i.e. the matrix K in the lecture notes, how can we estimate the horizontal and vertical directions of the observed poster (in the camera coordinate frame) ?

4 Reconstruction (4 points)

q1 and q2 are two image observations of the 3D point Q. Due to the noise, these points do not correspond to the exact projection of Q. As a result, the viewing lines of q1 and q2 do not intersect in 3D.

1. Describe an approach to compute, along the viewing of q1, the depth λ_1 of the 3D point on this line that is closest to the viewing line of q2.
2. What would then be the 3D point closest to both viewing lines ?
3. Is there a closed form solution for λ_1 or do we need to solve a system of equations ?

5 3D Modeling (2 points)

1. Considering a point on a shape silhouette in an image, can we tell whether the corresponding point in 3D is a convex, concave or saddle point on the observed shape ?
2. Depending on the number of viewpoints available we can perceive a scene in 2D or 3D using adapted displays. How many viewpoints are required for 3D perception on a stereo screen ? in a head mounted display ? explain why they differ.