Computer Vision - 3D Modeling

All printed documents are allowed. The different sections below are independent. Answers must be justified.

1 Projective Geometry (8 points)

1. Demonstrate that between any point basis of $\mathcal{P}^n$ and the canonical basis of $\mathcal{P}^n$ there exists a linear transformation represented by a non-singular matrix A which is defined up to a scale factor (theorem 2).

2. As a consequence, show that 2 point basis of $\mathcal{P}^n$ are related by a homography (theorem 3)

3. Show that parallelism is preserved under affine transformations.

4. Show that that that intersection of two lines is preserved under projective transformations.

2 Image Mosaics (6 points)

A camera acquires images while rotating around its projection center.

1. Show that the transformation between any 2 such images is a 3×3 homography.

2. How can we compute this homography given 2 images (detail your answer) ?

3. In order to build the mosaic, all images are projected onto the same planar domain, e.g. a plane or a cylinder. Sketch an algorithm for doing so.

3 Surface modeling (6points)

We are given a 3D modeling system composed of n cameras. Cameras are fixed and calibrated and we consider the 3D modeling process using information extracted from the images.

1. Assume a finite number n of silhouettes are availabe, is the topology (i.e. gender and number of connected components) of the corresponding visual hull necessarly equal to the topology of the observed object ?
2. Propose an algorithm that computes the visual hull based on a space discretization into voxels (the algorithm will be sketched only).
3. The object under consideration is not completely observed by all cameras. Modify your algorithm, if necessary, so that it handles also this situation.
4. In the case of a Lambertian surface can we improve such model ? how ?