

3D Modeling using multiple images

Exam January 2008

All documents are allowed. The different sections below are independent. Answers should be justified.

1 3D Reconstruction – A Robust Approche

Consider a set of n images, whose projection matrices, i.e. the intrinsic and extrinsic parameters, are all known. Consider points $\mathbf{q}_1 \cdots \mathbf{q}_n$, where \mathbf{q}_i is a point in the i th image. These n points are supposed to match one another, i.e. they are supposed to be projections of the same 3D point. However, there may be outliers among them.

The goal of this question is to reconstruct the 3D point. Sketch an approach for doing this, that is robust to these outliers. It is not required to give formulas, but all steps of the approach should be clearly described (if one step corresponds exactly to a method described in the course notes, then it is enough to make a reference to this).

2 3D Reconstruction from Two Images – An Alternative Method

During the lecture, a 3D reconstruction method has been explained. The goal of this question is to develop an alternative method, for the case of two images.

Suppose that the intrinsic and extrinsic parameters of the two images are known. Let \mathbf{q}_1 and \mathbf{q}_2 be matching points in the two images. As always, these points are supposed to be noisy, i.e. they do not correspond to the exact projections of the original 3D point.

For the 3D reconstruction method to be developed, we propose to impose that the 3D point \mathbf{Q} lies on the line of sight associated with point \mathbf{q}_1 . The point \mathbf{Q} can thus be parameterized by a single variable, say λ .

Write down this parameterization. Describe how to compute λ , using the point \mathbf{q}_2 on the second image and the knowledge of intrinsic and extrinsic parameters.

3 Point matching for 3 Images

During the lecture, two key concepts for matching **pairs** of images have been explained: use of a geometric constraint given by the epipolar geometry and comparison of image windows in order to detect matching points (by computing e.g. a correlation score).

One of the problems when using only 2 images for matching is the possibility of obtaining a wrong solution: for a point in the first image, there may exist several points on the epipolar line in the second image which look similar to it.

This problem may be avoided by using more than 2 images. Here, we consider the case of 3 images. We suppose to know the epipolar geometry for each of the 3 pairs of images that may be formed. The goal is to find, for a point q_1 in the first image, the matching points q_2 and q_3 in the other two images.

Sketch a method for doing this. The method should benefit from the fact that 3 images are used, in order to avoid the above mentioned problem existing for the 2-image case.

Advice: It is not required to develop formulas; it is enough to describe the method using words, and if applicable, to refer to the course notes for details which they already contain.

4 Question Linked to Image Mosaics

Let us put in the same setting as the one considered for the generation of image mosaics: a camera acquires images while rotating about its optical center. Consider the following special case. The camera takes **two** images. Between the two images, it carries out a rotation about the X axis:

$$R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \gamma & -\sin \gamma \\ 0 & \sin \gamma & \cos \gamma \end{pmatrix}$$

As for the intrinsic parameters of the camera, we suppose that they correspond to a simplified calibration matrix whose only unknown is the focal length α :

$$K = \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

During the lecture, it has been shown that there exists a projective transformation (an homography) that links the two images, and that it can be estimated from 4 or more point matches.

The goal of this question is to estimate the homography from a single point match. To do so, write down the homography H in terms of the two unknowns, the rotation angle γ and the focal length α . Then, develop a method (formulas) for computing these two unknowns, from a single point match (\mathbf{q}_1 in the first image and \mathbf{q}_2 in the second one).

5 Projective Geometry

1. What are the benefits of the projective geometry with respect to the Euclidean Geometry ?
2. How do we go from a projective space to an affine space ?
3. Explain the duality principle in a projective space ?
4. What is an invariant for a group of transformation and why does affine transformations preserve parallelism ?

6 Surface modeling

We are given a 3D modeling system composed of n cameras. Cameras are fixed and calibrated and we consider the 3D modeling process using information extracted from the images.

- We first assume that the silhouettes $S_{i \in [1..n]}$ of an observed object, and only the silhouettes, are available:
 1. What kind of model can we obtain ?
 2. When n goes to infinity what becomes the model ?
 3. Propose an algorithm that computes this model based on space discretization into voxels (the algorithm will be sketched only).
 4. The object under consideration is not completely observed by all cameras. Modify your algorithm, if necessary, so that it handles also this situation.
- We consider secondly the photometric information:
 1. Can we improve the models computed before using geometric information only ? if yes how ?
 2. In the case of a Lambertian surface and if we assume the calibration to be exact can we obtain a model of the observed object which is exact up to the discretization ?
 3. Voxel based models present a tradeoff between precision and complexity. Explain how the grid based algorithm could be modified in order to improve this tradeoff.
- Images are acquired, without any loss, by the cameras with a frequency f . Do those cameras necessarily need to be synchronized in the acquisition process ?

7 Motion modeling

Modeling motion consists in obtaining, from images, information on the motion of a moving object in the scene.

1. What are the main differences with shape modeling and what is the interest of such information ?
2. Explain (sketch) how the Vicon system works and identify the principle steps required, once images are acquired, to estimate motion with such systems.